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Broken Symmetry and the Accelerated Observer [1]

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Abstract

Spontaneous symmetry breaking affords an interesting probe of the phenomenon of "acceleration radiation" in general relativistic quantum field theory and raises a peculiar paradox. Accelerated observers detect the presence of particles in the vacuum in a thermal distribution with a temperature proportional to the proper acceleration. Nonetheless, by ever increasing the temperature (acceleration) a broken symmetry cannot be restored without violating general covariance. How does the accelerated observer interpret this outcome dynamically?

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I. Introduction

Coordinate systems possessing horizons lead to certain ambiguities in the definition of a quantum field theory [1, 2]. One may consider a collection of observers comoving in such a coordinate system (tacked down to some fixed values of the spatial coordinates; such observers cannot all be freely falling; examples include the comoving observers in Schwarzschild, static deSitter and Rindler coordinates). We attempt to define a Hamiltonian, H_h, which propagates the Schroedinger wave-functional of the quantum field theory in the accelerated observer's coordinate time (we find this to be the simplest and most conceptual approach, though any conventional formulation of field theory will due; however, there are subtleties with path intgrals since the initial time and final time surfaces in these systems always overlap). However, the singularity in the coordinate system on the horizon translates into ambiguities in the definition of the Hamiltonian density at the horizon and hence the Hamiltonian integral across the horizon. The ground state of the Rindler Hamiltonian is the "Unruh vacuum" and is seen to have *lower* energy than the Minkowski ground state.

These ambiguities are related to the Casimir effect. If one artificially severs continuity normal to some plane in flat space, i.e. neglect the $\nabla\phi \cdot \nabla\phi$ terms in the Hamiltonian on this surface, then the groundstate of the field theory will have different energy than the usual Minkowski vacuum. This owes to the singular field configurations (whose normal derivatives to the plane are nonexistent) which previously had zero amplitude of being found in the vacuum now becoming active and establishing a new groundstate. This is similar to the familiar Casimir effect in which parallel plane conducting surfaces experience a net force due the expulsion of vacuum zero-point fluctuations which are inconsistent with conducting boundary conditions. In fact, this is effectively what happens in the singular coordinate system and leads to the Unruh vacuum being physically distinct from the Minkowski case. The formal resemblance of the Unruh matrix elements to those of an infinite plane conductor in Minkowski space with *Dirichlet* boundary conditions are striking.

Rindler coordinates are defined in flat space and describe a comoving ensemble of accelerated observers and are given by: [3, 4]:

$$t = a^{-1}\, e^{a\xi} \sinh(a\eta) \tag{1.1}$$

$$x = a^{-1}\, e^{a\xi} \cosh(a\eta) \qquad\qquad (x > 0) \tag{1.2}$$

$$x_\perp = x'_\perp \tag{1.3}$$

where $(-\infty < \eta, \xi < \infty)$. We will presently restrict our attention to the "right hand wedge" corresponding to $x > 0$, though it is straightforward to extend the results to the double wedge case. Eq.(1.3) describes observers of fixed ξ accelerating with proper acceleration given on the $t = \eta = 0$ time slice by $ae^{-a\xi} = 1/x$, and elapsed proper time $\eta e^{a\xi}$. The metric in Rindler coordinates is given by:

$$ds^2 = e^{2a\xi}(d\eta^2 - d\xi^2) - dx'^2_\perp \tag{1.4}$$

Presently we will adopt a covariant functional Schroedinger description of the system as developed in ref.(5). We refer the reader to ref.(5) for the formal details. An equivalent approach might be to construct the appropriate Green's functions [6] in the Unruh vacuum and extract local matrix elements from these.

The true physical vacuum is always the usual Minkowski one, and operator matrix elements simply transform covariantly to the accelerating frame. Thus, since $\langle : \phi^2 : \rangle$ is zero (upon renormalization), it will always be measured to be zero by any observer. The Minkowski vacuum in Schroedinger picture is given by a gaussian wave-functional of the form:

$$\Psi_M = \exp\left\{ -\tfrac{1}{2} \int dk_x d^{d-1}k_\perp \mid \alpha(k_x, k_\perp) \mid^2 \sqrt{k_x^2 + k_\perp^2 + m^2} \right\} \tag{1.5}$$

where we have represented the wave–functional in momentum space. For example, with a plane conducting wall at $x = 0$ and Dirichlet boundary conditions we would have the expansion:

$$\phi(x) = \int_0^\infty dk_x \sqrt{\frac{2}{\pi}} \int \frac{d^{d-1}k_\perp}{(2\pi)^{\frac{d-1}{2}}} e^{ik_\perp \cdot x_\perp}\ \alpha(k_x, k_\perp) \sin k_x x \tag{1.6}$$

(recall that in Schroedinger picture the fields are generalized coordinates and carry no time dependence, which is carried by the wavefunctional; we do not indicate the time dependence which is irrelevant presently; we are free to go to the Fouriecoefficients as the coordinates of the system). The same field configuration can be represented in the right hand Rindler wedge in terms of massive $d+1$ dimensional Rindler modes as [5]:

$$\phi(x) = \int dk_z \frac{d^{d-1}k_\perp}{(2\pi)^{\frac{d-1}{2}}} \beta(k, k_\perp)\ e^{ik_\perp \cdot x_\perp}\ R_{k_z}(\varsigma) \tag{1.7}$$

where:

$$R_p^{k\perp}(\varsigma) = \frac{1}{\pi}\left(\frac{2p}{a}\sinh\frac{\pi p}{a}\right)^{\frac{1}{2}} K_{\frac{ip}{a}}(a^{-1}e^{a\varsigma}\sqrt{k_\perp^2+m^2}) \tag{1.8}$$

These modes diagonalize the Rindler Hamiltonian. We may then represent the Minkowski wave-functional as a gaussian in the coefficients, $\beta(k)$ (this is equivalent to a Bogoliubov transformation in the usual formalism and is given in ref.(5) as):

$$\Psi_M = \exp\left\{-\frac{1}{2}\int\ dk_z d^{d-1}k_\perp \mid \beta(k,k_\perp)\mid^2 k_z \coth\left(\frac{\pi k_z}{2a}\right)\right\} \tag{1.9}$$

This is not a groundstate of the Hamiltonian written in Rindler coordinates since the width for each mode has an extra factor of $coth(\frac{\pi k_z}{2a})$. Indeed, Ψ_M now appears as a state full containing a Bose gas of Rindler particles (these are particles as defined relative to the Rindler Hamiltonian; the full double wedge Minkowski case is similar [5]) with a universal temperature of $T = \frac{a}{2\pi}$.

The groundstate of the Rindler Hamiltonian is the "Unruh" vacuum and is given by:

$$\Psi_U = \exp\left\{-\frac{1}{2}\int dk_z d^{d-1}k_\perp \mid \beta(k,k_\perp)\mid^2 \mid k_z \mid\right\} \tag{1.10}$$

Since this state is clearly different than the Minkowski groundstate and since a given Hamiltonian can have only one groundstate, it follows that the Rindler Hamiltonian is different than the Minkowski Hamiltonian. This difference arises due to the singular structure of the Rindler coordinates associated with the horizon.

II. Operator Expectation Values

If we compute local operator matrix elements, such as $\langle\phi^2\rangle$ in the Unruh vacuum, we find that they are not covariant transforms of the same operator evaluated in the

Minkowski vacuum, but rather develop generally negative "thermal" corrections. For example, we show that $\langle\phi^2\rangle$ becomes $-\frac{T^2}{12}$ in an "high temperature limit" where T is the local Hawking temperature given in terms of the local proper acceleration (at $t=\eta=0$ we have $T=\frac{1}{2\pi x}$). Thus, *it is the difference between the value of the operator in the Minkowski vacuum and that in the Unruh vacuum which appears as a positive thermal effect.*

Consider now the matrix element:

$$\left\langle\phi(x+\frac{\epsilon}{2})\phi(x-\frac{\epsilon}{2})\right\rangle = \int dk_x dp_x d^{d-1}k_\perp d^{d-1}p_\perp\ 2^{2-d}\pi^{-d} \left\{\sin k_x(x+\frac{\epsilon}{2})\sin k_x(x-\frac{\epsilon}{2})\right\} \cdot \langle\alpha(k_x,k_\perp)\alpha(p_x,p_\perp)\rangle\ e^{ik_\perp\cdot x_\perp + ip_\perp\cdot x_\perp} \tag{2.1}$$

The expectation value is to be taken in the wavefunctional of eq.(??). We have:

$$\langle\alpha(k_x,k_\perp)\alpha(p_x,p_\perp)\rangle = \int D\phi\ \Psi^*_M(\phi)\alpha(k_x,k_\perp)\alpha(p_x,p_\perp)\Psi_M(\phi) = \frac{\delta(k_x-p_x)\cdot\delta^{d-1}(k_\perp+p_\perp)}{2\left(k_x^2+k_\perp^2+m^2\right)^{\frac{1}{2}}} \tag{2.2}$$

Making use of various integral identities, including the d-dimensional solid angle, we arrive at the result [8]:

$$\left\langle\phi(x+\frac{\epsilon}{2})\phi(x-\frac{\epsilon}{2})\right\rangle = 2^{-d}\pi^{\frac{-1-d}{2}} \cdot\left\{\left(\frac{\epsilon}{2m}\right)^{\frac{1-d}{2}} K_{\frac{1-d}{2}}(m\epsilon) - \left(\frac{x}{m}\right)^{\frac{1-d}{2}} K_{\frac{1-d}{2}}(2mx)\right\} \tag{2.3}$$

We note that the UV singularity of the operator ϕ^2 resides in the Bessel functions with arguments $m\epsilon$. This result is, of course, equivalent to evaluating the Feynman propagator for spacelike interval with Dirichlet boundary conditions. We further note that in the Lorentz invariant vacuum without the presence of the wall at $x=0$ we obtain the familiar result:

$$\left\langle\phi(x+\frac{\epsilon}{2})\phi(x-\frac{\epsilon}{2})\right\rangle_{Lorentz} = 2^{-d}\pi^{\frac{-1-d}{2}}\left\{\left(\frac{\epsilon}{2m}\right)^{\frac{1-d}{2}} K_{\frac{1-d}{2}}(m\epsilon)\right\}$$

Clearly, the short distance singular part of eq.(2.3) is not influenced by the boundary conditions, and can be unambiguously subtracted in all coordinate systems (this corresponds to renormalizing the operator matrix element to be zero in the limit of zero acceleration).

The formalism developed in ref.(5) is covariant and we have verified explicitly as a check on the present calculation that if we reexpress the Minkowski vacuum, $\Psi_M(\phi)$ in terms of the Rindler modes and recalculate the $\langle \phi^2 \rangle$ we obtain the same result as in eq.(2.3) [8]. Hence, although the Minkowski vacuum appears to be full of particles, the local operator matrix elements are covariant and the acceleration radiation is in a sense fictitious.

It is of interest however to evaluate the matrix element $\langle \phi^2 \rangle$ in the physically distinct Unruh vacuum. This is formally equivalent to the preceding analysis, but involves a nontrivial evaluation of a resulting Kontorovich-Lebedev transformation. This is discussed in ref.(8) We obtain the result:

$$\begin{aligned}\left\langle \phi(x+\frac{\epsilon}{2})\phi(x-\frac{\epsilon}{2}) \right\rangle_U = 2^{-d}\pi^{\frac{1+d}{2}} \Bigg\{ \left(\frac{\epsilon}{2m}\right)^{\frac{1-d}{2}} K_{\frac{1-d}{2}}(m\epsilon) \\ - 2\int_0^\infty \left\{ \frac{\sqrt{x_1^2+x_2^2+2x_1x_2\cosh\omega}}{2m} \right\}^{\frac{1-d}{2}} \\ \cdot K_{\frac{1-d}{2}}\left(m\sqrt{x_1^2+x_2^2+2x_1x_2\cosh\omega} \right) \frac{d\omega}{\pi^2+\omega^2} \Bigg\}\end{aligned} \tag{2.5}$$

The second term on the right–hand side is nonsingular in the $\epsilon \to 0$ limit and we thus are led to the result:

$$\begin{aligned}\left\langle \phi(x)^2 \right\rangle_U = 2^{-d}\pi^{-\frac{1+d}{2}} \Bigg\{ \left(\frac{\epsilon}{2m}\right)^{\frac{1-d}{2}} K_{\frac{1-d}{2}}(m\epsilon) \\ - 2^{\frac{d+1}{2}} m^{d-1} \int_0^\infty \frac{d\omega}{\pi^2+\omega^2}\, Q^{\frac{1-d}{2}}\, K_{\frac{1-d}{2}}(Q) \Bigg\}.\end{aligned} \tag{2.6}$$

where:

$$Q = \sqrt{2}mx\,\sqrt{(1+\cosh\omega)}. \tag{2.7}$$

Thus, the singular structure is identical to that obtained above for the Minkowski, an Dirichlet results. The finite corrections are negative definite and analogous to those obtained for the Dirichlet case. This is not unreasonable mathematically since the Rindler mode functions oscillate infinitely as they approach the horizon, while all normalization integrals have effectively a compact support. As such, we are implicitly forcing the field configuration of eq.(1.7) to vanish at the horizon by our normalization conventions and this in turn yields the result of eq.(2.6) not unlike the Dirichlet result.

Eq.(2.6) yields a more striking result when we consider it in a specific case. Let us specialize to $d = 3$ corresponding to $3 + 1$ dimensional spacetime. We further consider the limit of small x, the "high acceleration" limit. Throwing away the singular ϵ-terms we find the leading behavior:

$$\left\langle :\phi(x)^2: \right\rangle_U \to -\frac{1}{4\pi^2 x^2}\int_0^\infty \frac{d\omega}{(1+\cosh\omega)(\pi^2+\omega^2)} = -\frac{T^2}{12} \tag{2.8}$$

where we define the *local Hawking Temperature* $T(x) = \frac{1}{2\pi x}$ where the local proper acceleration is given by $\frac{1}{x}$ (we have used the integral eq.(??) to obtain this latter result as well as the small argument limit of the Bessel function $K_1(x)$).

This result is *minus* the usual thermal correction to the operator ϕ^2 as is easily verified by computing the expectation value with the thermal density matrix. It suggests that locally the Minkowski vacuum expectation value, which is zero upon subtraction, is "hot" by an amount $\frac{T^2}{12}$ when compared to the Unruh result. Nonetheless, there is no conflict with general covariance because the result in Minkowski space is invariant, i.e. zero transforms into zero. *It would be incorrect to conclude that an accelerating observer measuring* $\langle\phi^2\rangle$ *obtains a thermal result of* $\frac{T^2}{12}$.

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